TELESCOPE MATH

Basic Math That Amateur Astronomers Need to Understand Telescopes

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OVERVIEW

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- Telescope Resolution
- Focal Ratio, f/#
- Magnification
- Exit Pupil Size
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- Field-of-View of Camera Sensor (For Astrophotographers)
- Summary

MOTIVATION

LEARNING THE BASIC MATH BEHIND THE TELESCOPE:

- Provides insight into optics, which is the fundamental enabler of astronomy and photography.
- Allows a person to use a telescope with understanding.
- Enables effective communication between fellow amateur astronomers themselves and the public.
- Informs the decision making process when buying a telescope or one of its optical accessories like an eyepiece.

THE CLASSIC PITFALL

The Walmart "Xmas" Special

$86.58!!!

Package Included:

- 1x Zoom Eyepiece 7.5mm - 22.5mm 31.7mm (1.25") SZ1
- 1x Eyepiece 31.7mm (1.25") 20mm
- 1x Eyepiece 31.7mm (1.25") 12.5mm
- 1x Eyepiece 31.7mm (1.25") 6mm
- 1x Eyepiece 31.7mm (1.25") 4mm
- 1x 1.5x Erecting Eyepiece 31.7mm (1.25")
- 1x Moon Filter 31.7mm (1.25")
- 1x Barlow-Lens 31.7mm (1.25")
- 1x Finder Scope 5x24
- 1x Sturdy, vertically adjustable aluminum tripod

OVER PROMISED PERFORMANCE WILL DISAPPOINT AND FRUSTRATE!

Specification:

- Aperture: 76mm (3")
- Focal Length: 700mm (27.6")
- Faintest discernable stars: 12.0M! (over 1,000,000 Stars visible!)
- Dawes Limit: 1.9 arc-seconds
- Focal Ratio: f/9
- Eyepieces: 31.7mm (1.25")
- Zoom Eyepiece: Steplessly adjustable focal length 7.5mm - 22.5mm
- Zoom Eyepiece: 31.7mm (1.25") socket
- Magnification normal: 4mm/175x, 6mm/117x, 12.5mm/56x, 20mm/35x
- Magnification with Barlow-Lens: 4mm/350x, 6mm/234x, 12.5mm/112, 20mm/70x
- Magnification with Zoom Eyepiece: 31x to 93x
- Magnification with Zoom Eyepiece and Barlow-Lens: 62x to 186x

https://www.walmart.com/ip/76mm-Aperture-Astronomical-Telescope-76700-AL-700mm-Focal-Length-Reflector-Travel-Scope-With-Tripod-For-Beginners-Students-Kids-Children/113430956

TELESCOPE RESOLUTION

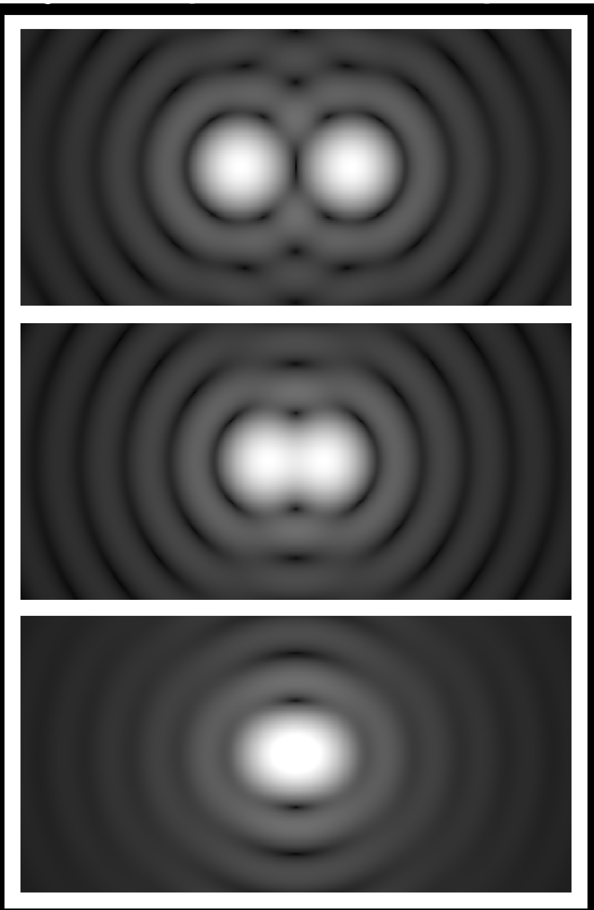
- Resolving power of an optical instrument is its ability to spatially separate far away, yet close objects, that are close together, in the resulting image.
- Many definitions for resolution criteria include:
  - Rayleigh: $\theta = 1.220\frac{\lambda}{D}$ (Line resolution of spectroscope)
  - Dawes: $\theta = 1.021\frac{\lambda}{D}$ (Telescope observation of double stars)
  - Abbe: $\theta = 1.000\frac{\lambda}{D}$ (Optical theory)
  - Sparrow: $\theta = 0.950\frac{\lambda}{D}$ (Laboratory visual experiments)

  where
    - θ is the angular separation between objects (see figure), in radians.
    - λ is the wavelength of the light from the objects, in meters.
    - D is the diameter of the primary optic (*i.e.*, lens or mirror), in meters.

TAKEAWAY:

- Diameter of a telescope's primary optic, D, controls resolution.
- Larger the optic's diameter, angular separation decreases → ability to see finer details increases.
  - Atmospheric turbulence eventually becomes limiting factor.
- Larger the optic's diameter, more light is collected → dimmer objects become brighter.

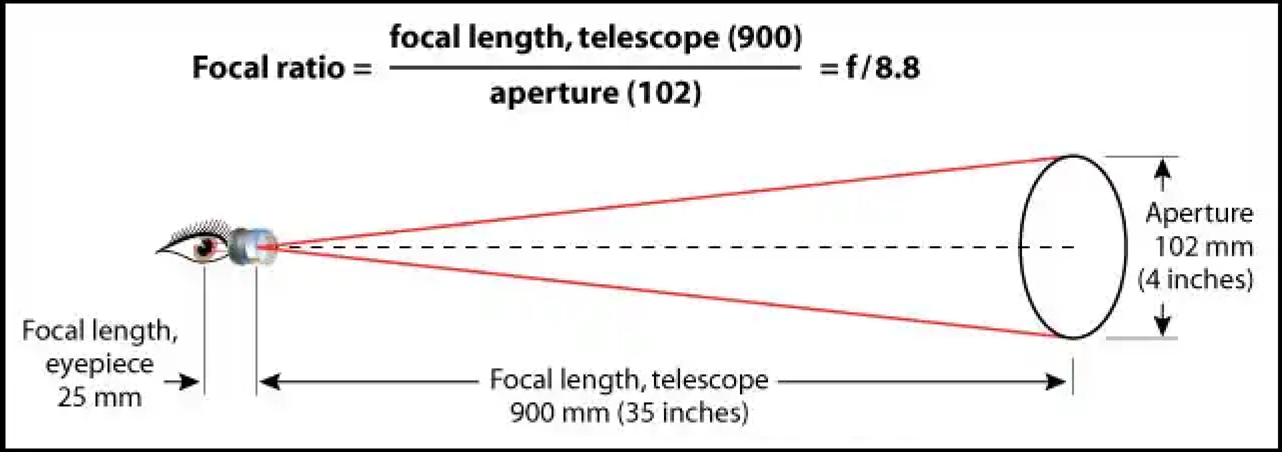
Angular Separation of Two Objects Represented as Airy Disks



FOCAL RATIO, F/#

- Focal ratio, *f/#*, or “f-stop” or “f-number” describes the limiting effect optics have on the imaging process.
  - Governed by the size of the unobstructed clear aperture in an optical system.
- Limiting diameter, which determines the amount of light reaching the imaging plane, is called the aperture stop.
  - For telescopes, the primary lens or mirror serves as the aperture stop.

$$f/\# = \frac{f_{primary}}{D} = \frac{focal\ length\ of\ primary\ optic}{diameter\ of\ primary\ optic}$$



FAST vs. SLOW TELESCOPES

- Smaller the f/#, more light enters, less time required for the exposure (i.e., “faster”).
  - Astrographic telescopes have f/4 or smaller.
- Larger the f/#, less light enters, more time required for the exposure (i.e., “slower”).
  - SCT primary mirror are f/10.
  - Hubble’s primary mirror is f/24.

MAGNIFICATION

- Magnification, M, is the increase in the angular size (apparent width) of an object in comparison to the naked eye size of the object.

$$M = \frac{f_{primary}}{f_{eyepiece}} = \frac{focal\ length\ of\ primary\ optic}{focal\ length\ of\ eyepiece}$$

- Magnification has limited utility, more may not be better!
- Magnification constricts the exit pupil of your telescope/eyepiece.

MAGNIFICATION LIMITS THE AMOUNT OF LIGHT ENTERING YOUR EYE!

- Maximum Magnification Rules of Thumb:
  - Typical degradation from atmospheric and/or optical imperfections: $$M_{max} = 30 \times D\ [inches]$$
  - Low degradation from atmospheric and/or optical imperfections: $$M_{max} = 60 \times D\ [inches]$$

EXIT PUPIL SIZE

- As stated previously, the aperture stop is the limiting diameter in the optical system that determines the amount of light reaching the imaging plane .
- **Image of the aperture stop** as seen from the eyepiece **is the exit pupil**.
- Exit pupil is the diameter of the "light pencil" that emerges from the eyepiece.
  - The pupil of fully dark-adapted human eye can dilate to about 7mm diameter, so **an exit pupil in excess of 7mm is passing more light than the eye can accept**.
  - As the exit pupil decreases **below 7mm, lack of light becomes the basic limiting factor** to what your eye can see.
  - Exit pupils of less than a 1.0 mm are troublesome because they can highlight blood vessels and floaters within the eye. Additionally, they magnify limiting diffractive effects caused by the telescope and atmosphere; the result is a very fuzzy image.

$$Exit\ Pupil = \frac{D}{M} = \frac{diameter\ of\ primary\ optic}{magnification}$$

(left) eyepiece exit window; (right) telescope exit pupil

TRUE FIELD-OF-VIEW

- Angular extent as seen through the exit pupil is the true field-of-view.

$$TFOV = \frac{AFOV_{eyepiece}}{M} = \frac{Eyepiece\ Apparent\ FOV}{Magnification}$$

- Most useful criteria when selecting/purchasing eyepieces.
  - Quantifies expectation as seen by the eye.

EYEPIECE COMPARISON WITH M42

EP Focal Length (left) = EP Focal Length (right)
Magnification (left) = Magnification (right)
Apparent FOV (left) < Apparent FOV (right)
True FOV (left) < True FOV (right)

EYEPIECE COMPARISON WITH M64

EP focal length (left) > EP focal length (right)
Magnification (left) < Magnification (right)
Apparent FOV (left) < Apparent FOV (right)
True FOV (left) = True FOV (right)

EXAMPLE: PURCHASING EYEPIECES

Telescope Model	**Telescope Type**	**INPUT: Aperture Size, *D* [inches]**	**INPUT: *f*/#**	**Aperture Size [mm]**	**Focal Length [mm]**	**Maximum Magnification Rules of Thumb (1)**				
						Low degradation from atmospheric and/or optical imperfections		Typical degradation from atmospheric and/or optical imperfections		
Meade AR-6	Achromatic Refractor	**6.0**	**8**	152.4	1219.2	60 x *D* [in]:	**360**	30 x *D* [in]:	**180**	

EYEPIECE SPECIFICATIONS & EXPECTED PERFORMANCE WITH TELESCOPE

Eyepiece Manufacturer	**Eyepiece Type**	**Barrel Diameter [inches]**	**Eyepiece Focal Length [mm]**	**Apparent FOV [°]**	**Magnification**	**Exit Pupil [mm]**	**Real FOV [°]**	**Real FOV [arcmin]**	**Real FOV [arcsec]**	**Cost**
Explore Scientific	100º Series	3	30	**100**	40.6	3.75	2.461	147.6	8858	$1,259.99
Meade	4000 Series Super Plössl	2	56	**52**	21.8	7.00	2.388	143.3	8598	$69.95
Garrett Optical	SWA	2	38	**69**	32.1	4.75	2.151	129.0	7742	$79.95

NOTE: Quoted eyepiece prices from 7 – 10 years ago (not current).

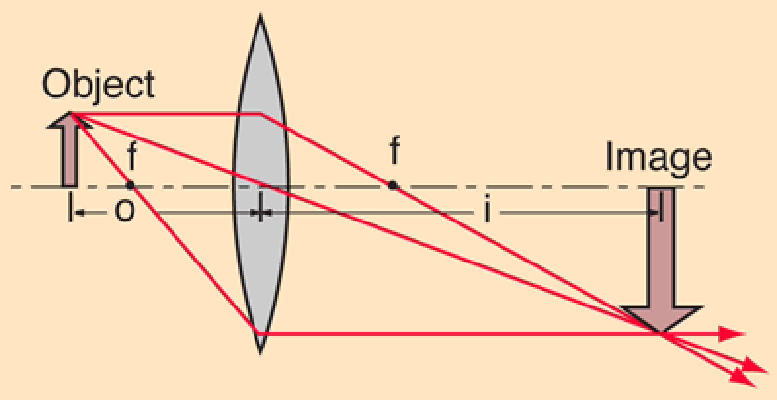
- For approximately the same Real FOV,
  - ExploreScientific provides the highest magnification but the smallest exit pupil for $1200.
  - Meade Plössl provides the lowest magnification but the largest exit pupil for $70.
  - Garrent SWA provides listed performance between the previous two for $80.
    - For the cost, the Garrett eyepiece has reasonable good performance overall.
    - A more compelling reason is needed to justify purchasing the ExploreScientific eyepiece; otherwise, the Garrett eyepiece would be a great eyepiece to purchase.

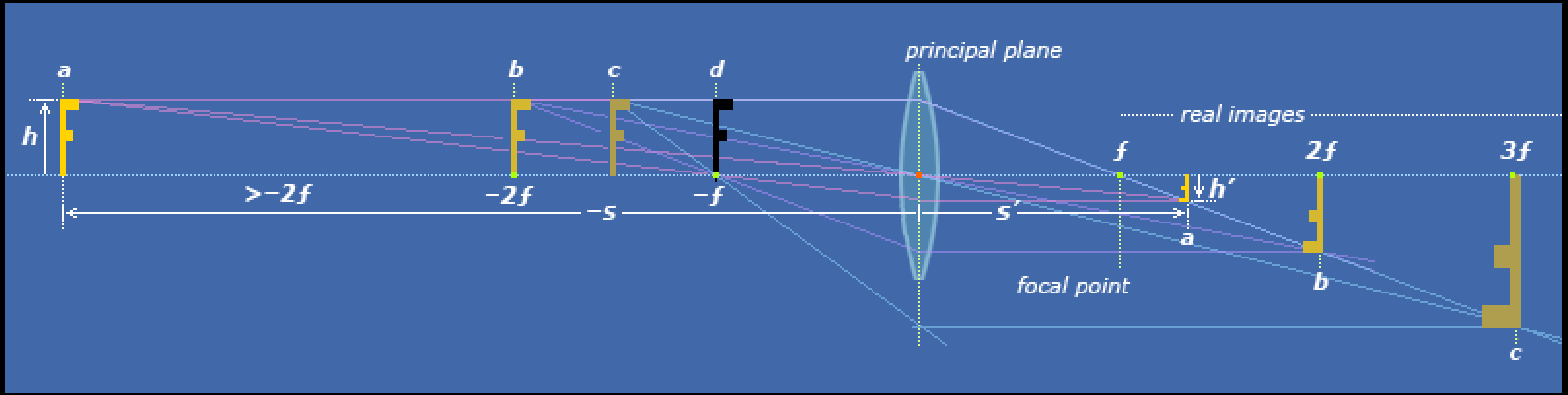
FIELD-OF-VIEW OF CAMERA SENSOR

- To this point, the discussion has only considered on how a visual observer is affected by the telescope/eyepiece combination.
- How do astrophotographers work with these parameters when there is no eyepiece (i.e. prime focal imaging)?
  - Telescope Resolution: No change in application or meaning.
  - Focal Ratio, f/#: No change in application or meaning.
  - Magnification: Specified by the telescope's focal length.
    - Short focal lengths correspond to small magnification.
    - Long focal lengths correspond to high magnification.
    - Adding a focal reducer to a long focal length telescope will create a shortened effective focal length; thereby decreasing magnification and widening the FOV
  - Exit Pupil Size: No longer applicable.
    - However, sufficiently small apertures or large f/#'s may produce a converging cone of light at the image plane that may be smaller than the camera sensor.

QUICK ASIDE: GEOMETRIC (RAY) OPTICS

- Geometrical optics, or ray optics, is a model of optics that describes light propagation, under certain circumstances and assumptions, in terms of rays.
- The image size and its location as defined with a “thin” lens are determined by using three principal rays from a single off-axis object point (see bottom left figure):
    1. A ray from the top of the object proceeding parallel to the centerline perpendicular to the lens. Beyond the lens, it will pass through the principal focal point.
    2. A ray through the principal focal point on the object side of the lens. It will proceed parallel to the centerline upon exit from the lens.
    3. A **chief ray** is an undeflected ray from the off-axis object point which passes through the center of the lens/aperture stop to the convergence of the two other rays.
- If an object is sufficiently far away from the optic, its image will appear at the focal point (see progression of image size in bottom right figure).

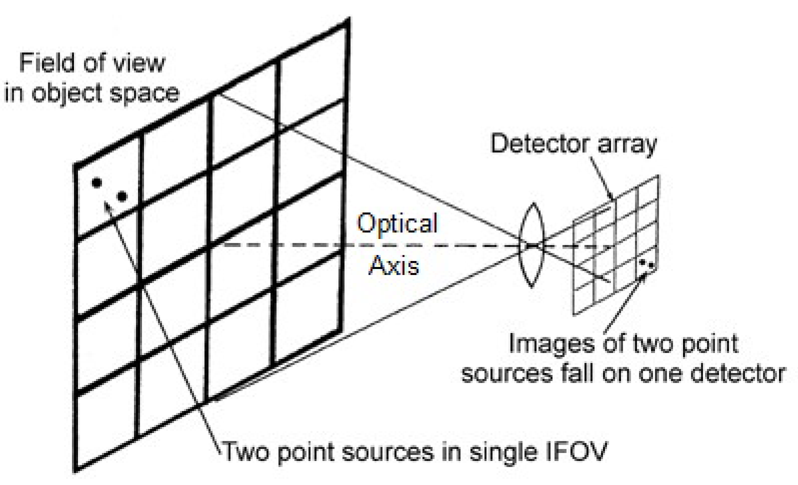




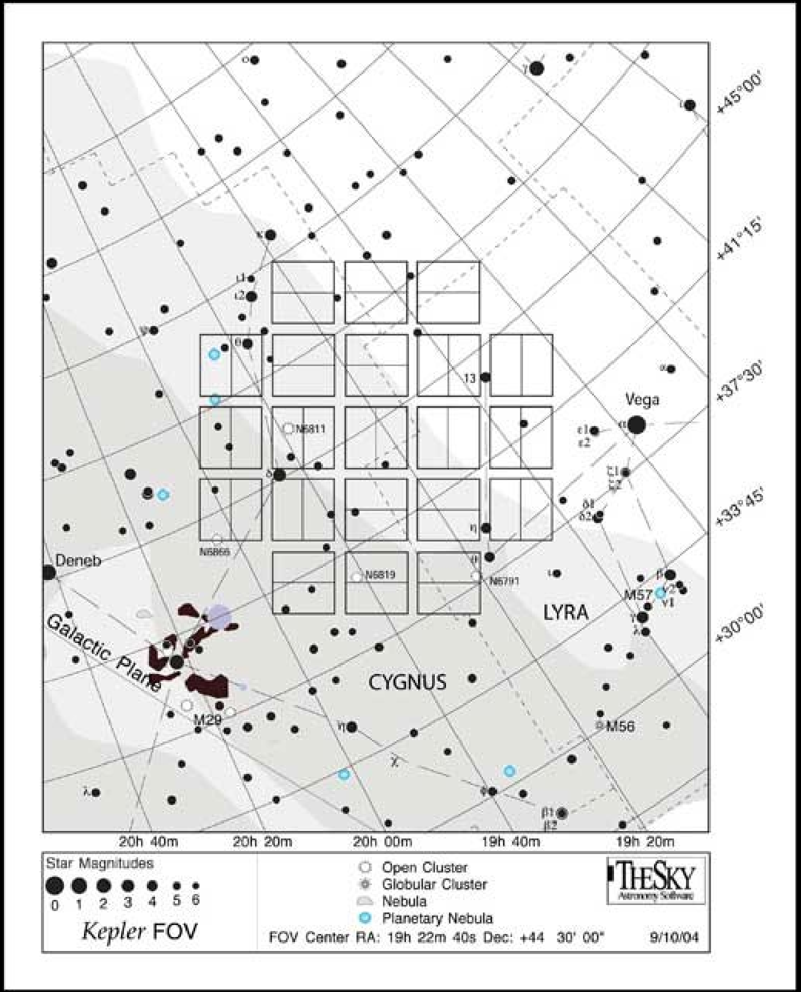
FIELD-OF-VIEW OF CAMERA SENSOR

- How can the field-of-view of a camera sensor be determined? → Focal Plane Projection.
  - Astronomical objects are located at “infinity” optically speaking (i.e., very long distance).
  - An object on the optical axis appears at the focal point.
  - Objects off the optical axis appear in a plane at the focal point.
  - The angles created by “off-axis” objects with respect to the optical axis are the same in “object” space as they are in “image” space inside a telescope.
- Projecting **chief rays** that start at the edge of the sensor (and individual pixels) allows for the FPA to be drawn onto the sky.
- **Instantaneous FOV** is the angular extent seen by a single pixel on a camera sensor. (*See figure to the right*.)

16 pixel (4 x 4) FPA



Projection of Kepler’s FPAs on to a Region in Cygnus



FIELD-OF-VIEW OF CAMERA SENSOR

- To calculate the IFOV and total FOV of a camera/telescope system, compile the following specs:
  - Horizontal and vertical dimensions of the camera's pixel ($H_{pix}\ x\ V_{pix}$) in [μm].
  - Number of horizontal and vertical pixels on the camera's sensor ($N_{pix,H}\ x\ N_{pix,V}$).
  - Focal length of the telescope connected to the camera ($f_{primary}$) in [mm]
- Convert the focal length [mm] and pixels [μm]. to meters, [m]

$$f_{primary}[m] = \frac{1[m]}{1000[mm]} f_{primary}[mm] \Rightarrow \boldsymbol{f_{primary}[m] = 0.001 f_{primary}[mm]}$$

$$H_{pix}[m] = \frac{1[m]}{1{,}000{,}000[\mu m]} H_{pix}[\mu m] \Rightarrow \boldsymbol{H_{pix}[m] = 0.000001 H_{pix}[\mu m]}$$

- Calculate the IFOV of a camera/telescope system (use small angle approximation):

$$IFOV_H = \theta \approx tan\theta = \frac{H_{pix}[m]}{f_{primary}[m]}[rad] = \frac{0.000001 H_{pix}[\mu m]}{0.001 f_{primary}[mm]} \times \frac{180[deg]}{\pi} \times \frac{3600[arcsec]}{1[deg]}$$

- Therefore, the horizontal and vertical IFOV or a camera's pixel is:

$$\boldsymbol{IFOV_H[arcsec] = 206.2648 \frac{H_{pix}[\mu m]}{f_{primary}[mm]}} \text{ and } \boldsymbol{IFOV_V[arcsec] = 206.2648 \frac{V_{pix}[\mu m]}{f_{primary}[mm]}}$$

FIELD-OF-VIEW OF CAMERA SENSOR

- To calculate camera sensor's total FOV, use the IFOV values along with the number of pixels that make up the sensor:

$$\boldsymbol{FOV_H[arcsec] = N_{pix,H} \times IFOV_H[arcsec]}$$ and $$\boldsymbol{FOV_V[arcsec] = N_{pix,V} \times IFOV_V[arcsec]}$$

- To convert from [arcsec] to [deg],

$$[deg] = \frac{1[deg]}{3600[arcsec]}[arcsec]$$

- Using the values for $IFOV_{H/V}$ and $FOV_{H/V}$ along with the angular dimensions of celestial objects, you can determine:
  - Whether the object is too big for your telescope/camera setup.
  - The number pixels across the object (e.g., a planet, cluster, nebula, galaxy, etc).

EXAMPLE CALCULATION: CAMERA SENSOR FOV

- Telescope: Meade AR-6 Achromatic Refractor, f/8, 152-mm aperture
    - Telescope focal length = (8)(152 mm) = 1216 mm
- Camera: Celestron Skyris 445C (Planetary Imager)
    - Total resolution: 1280 by 960 pixels
    - Pixel size: 3.75 by 3.75 μm
- Since the horizontal and vertical pixels dimensions are the same, $IFOV_H = IFOV_V$

$$IFOV[arcsec] = 206.2648 \frac{H_{pix}[\mu m]}{f_{primary}[mm]} = 206.2648 \frac{3.75[\mu m]}{1216\ [mm]} = \mathbf{0.636[arcsec]}$$

- The camera FOV is therefore:

$$FOV_H = N_{pix,H} \times IFOV_H = 1280 \times 0.636 = \mathbf{814[arcsec]}$$

$$FOV_V = N_{pix,V} \times IFOV_V = 960 \times 0.636 = \mathbf{610[arcsec]}$$

- At opposition, Jupiter's angular size is 50.59 [arcsec]; therefore, this telescope/camera setup will image the entire planet using 80 pixels across its diameter.

SUMMARY

- A significant amount of understanding about telescopes, eyepieces, binoculars, and cameras can be gained by using these simple concepts and equations.
- Discussion highlights that "optimal" telescope performance is a series of trade-offs based on what the amateur astronomer wants to accomplish.
- These simple mathematical tools are meant to empower amateur astronomers as they explore this wonderful hobby.
  - Provides operational understanding for how basic astronomical equipment works including the ability to troubleshoot problems.
  - Helps amateur astronomers make informed purchasing decisions about equipment.
  - Enables them to mentor and help new/budding/young amateurs just starting out.
  - Serves as a stepping stone for more complex aspects to astronomy, optics, and physics…

…How far down the rabbit hole you go is up to you!

QUESTIONS?